

COMPUTATION OF THE MITTAG-LEFFLER FUNCTION

$E_{\alpha,\beta}(z)$ AND ITS DERIVATIVE

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*Dedicated to Francesco Mainardi,
Professor at the University of Bologna,
on the occasion of his 60-th birthday*

Abstract

In this paper algorithms for numerical evaluation of the Mittag-Leffler function

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta + \alpha k)}, \quad \alpha > 0, \beta \in \mathbf{R}, z \in \mathbf{C}$$

and its derivative for all values of the parameters $\alpha > 0$, $\beta \in \mathbf{R}$ and all values of the argument $z \in \mathbf{C}$ are presented. For different parts of the complex plane different numerical techniques are used. In every case we provide estimates for accuracy of the computation; numerous pictures showing the behaviour of the Mittag-Leffler function for different values of the parameters and on different lines in the complex plane are included. The ideas and techniques employed in the paper can be used for numerical evaluation of other functions of the hypergeometric type. In particular, the same method with some small modifications can be applied for the Wright function which plays a very important role in the theory of partial differential equations of fractional order.

Mathematics Subject Classification: 33E12, 65D20, 33F05, 30E15

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